Dirichlet-Point processes

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Introduction

- Every minute:

400h of video
350 000 tweets
500 000 comments
4 200 000 searches

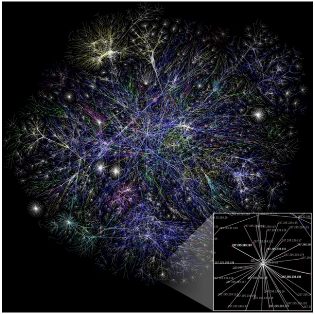


Figure 1: Snapshot of the internet (Wikipedia)

Motivation

- Every minute:

400h of video
350 000 tweets
500 000 comments
4 200 000 searches

- How to make sense out of *that*?

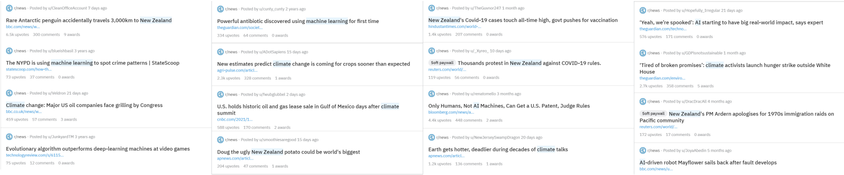


Figure 2: A typical stream from r/news

Motivation

- Every minute:

  400h of video
  350 000 tweets
  500 000 comments
  4 200 000 searches

- How to make sense out of *that*?
  → Hidden semantic links

Figure 2: A typical stream from r/news – with topics

Available information

- Main clues:
  - Textual information

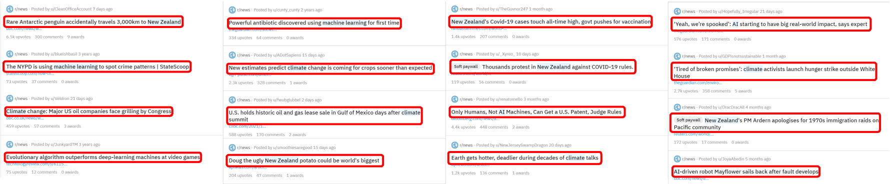


Figure 3: We can use textual information

Available information

- Main clues:
  - ◇ Textual information
  - ◇ Temporal information

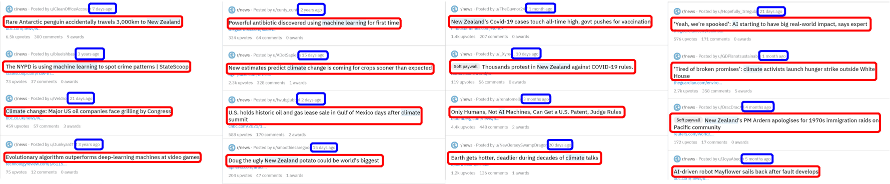


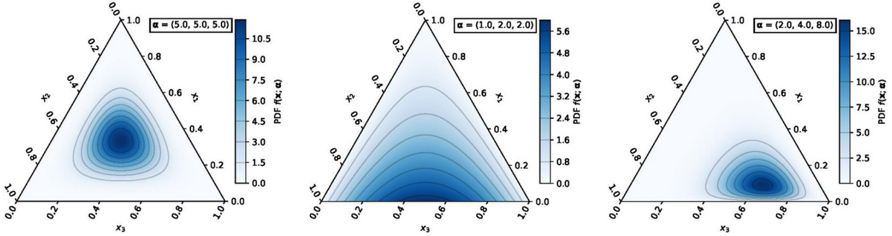
Figure 3: We can use textual information and temporal information

Documents stream

- The data is therefore a documents stream

Dirichlet process

- Dirichlet processes fit to consider streams as inputs
- Dirichlet distribution: $\vec{X} \sim Dir(\alpha)$ s.t. $\sum_k X_k = 1$
- Often used as a prior distribution in Bayesian clustering
  - Typically X_k is the probability to belong to cluster k
- Can be represented in several ways:
  - Stick-breaking process
  - Polya-Urn process
  - Chinese restaurant process



Chinese restaurant process

- Chinese Restaurant Process:

$$CRP(C_i = c | C_1, C_2, ..., C_{i-1}, \alpha) = \begin{cases} \frac{N_c}{\alpha+N} & \text{if c = 1, ..., K} \\ \frac{\alpha}{\alpha+N} & \text{if c = K+1} \end{cases}$$

Handling a stream of documents

- Chinese Restaurant Process:

$$CRP(C_i = c|C_1, C_2, ..., C_{i-1}, \alpha) = \begin{cases} \frac{N_c}{\alpha+N} & \text{if c = 1, ..., K} \\ \frac{\alpha}{\alpha+N} & \text{if c = K+1} \end{cases}$$

- Useful for sequential modeling (explicit posterior at each step, allows Gibbs sampling)

$$\underbrace{P(n^{th} obs = c|D, history)}_{Posterior} \propto \underbrace{P(D|n^{th} obs = c)}_{Likelihood} \times \underbrace{P(n^{th} obs = c|history)}_{CRP \;\; prior}$$

- Hypothesis: “rich-get-richer”

Variants

- Variants of DP exist:
  - ◇ Uniform process [Wallach et al., 2010]
  - ◇ Pitman-Yor process [Pitman and Yor, 1997]
  - ◇ Hierarchical Dirichlet process [Teh et al., 2006]
  - ◇ Nested Dirichlet process [Rodríguez et al., 2008]

→ Most exhibit "rich-get-richer" property

→ All consider counts, none consider temporal dimension

Modeling time as a continuous variable

- Time often "modeled" by sampling observations (DTM [Blei and Lafferty, 2006], TOT [Wang and McCallum, 2006], RCRP [Ahmed and Xing, 2008], DDCRP [Blei and Frazier, 2010], etc.)
  - ◇ Problems: how to slice data, which sampling function use, how to weight observations, etc.
- Whole literature modeling time explicitly: point processes

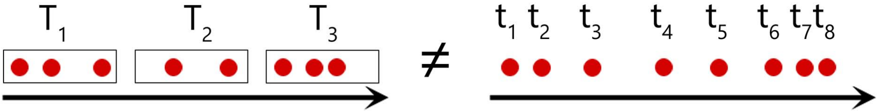


Figure 4: Data sampling/slicing is an approximation

Poisson process

- Poisson processes are characterized by an **intensity** λ.
  - $P(\mathbb{N}(t) = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$ = probability for n events to happen within a time t
- Instantaneous PDF of **one** event (or inter-arrival time PDF):

$$f(t) = \frac{P(\mathbb{N}(t) = 1)}{t} = \lambda e^{-\lambda t}$$

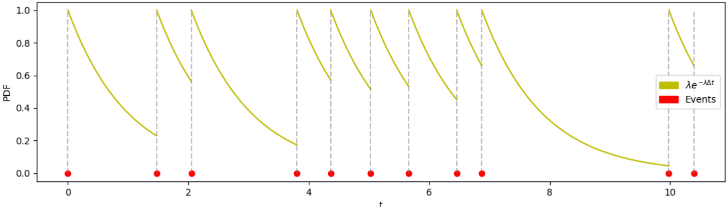


Figure 5: Could model radioactive decay events of atoms whose half-life is 1

Non-homogeneous Poisson process

- $\lambda(t)$ is a function
- $\lambda(t) = \lim_{\Delta t \to 0} \frac{P(\mathbb{N}(t+\Delta t)-\mathbb{N}(t)=1)}{\Delta t}$

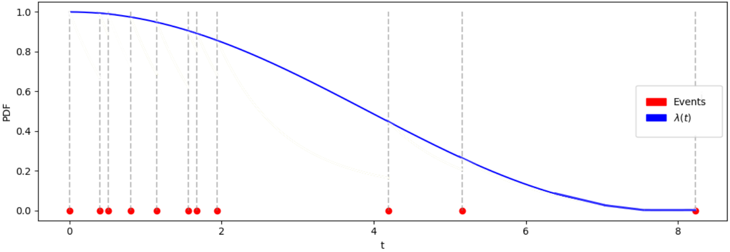


Figure 6: Could model cars arrival at gas station throughout a day

Hawkes process

- Hawkes processes: $\lambda(t)$ depends on past events $\mathcal{H}_t = \{t_i | t_i < t\}$
  $\rightarrow$ "Self-exciting process"
- Typically: $\lambda(t) = \lambda_0 + \sum_{t_i \in \mathcal{H}_t} \phi(t - t_i)$

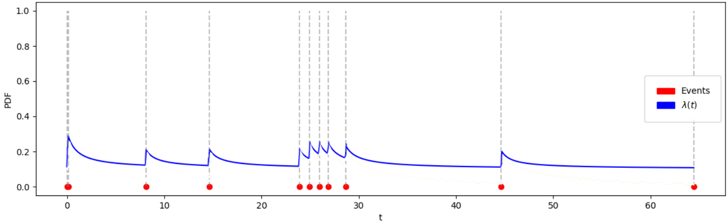


Figure 7: Could model online posting dynamics

Inference

- Log-likelihood $\ell(\lambda)$ fit for data streams:

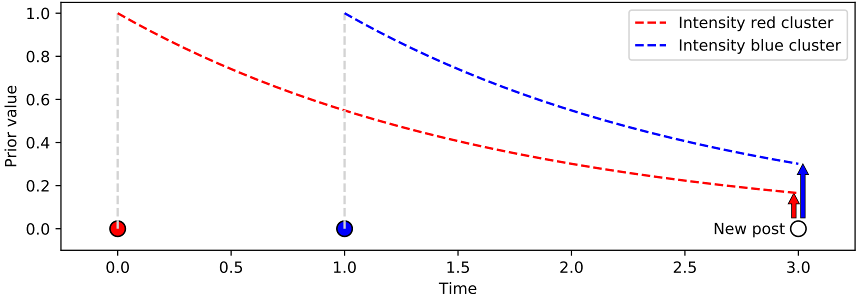
$$\ell(\lambda) = -\int_{t_0}^{t_N} \lambda(t)dt + \sum_{t_i<t_N} \log\lambda(t_i) = \log\lambda(t_1) - \int_{t_0}^{t_1} \lambda(t)dt$$
$$+ \log\lambda(t_2) - \int_{t_1}^{t_2} \lambda(t)dt$$
$$+ \dots$$
$$+ \log\lambda(t_N) - \int_{t_{N-1}}^{t_N} \lambda(t)dt$$

- Convex for certain shapes of $\lambda(t)$ (exp, ray, PL, Gaussian, ...).

Dirichlet-Hawkes process

- [Du et al., 2015]: Dirichlet-Hawkes prior (Bayesian inference)
- Merges Dirichlet priors and Hawkes processes

$$P(\text{cluster}|\text{text, time, H}) \propto \underbrace{P(\text{text}|\text{cluster})}_{\substack{\text{Textual likelihood} \\ \text{(Dirichlet-Multinomial)}}} \times \underbrace{P(\text{cluster}|\text{time, H})}_{\substack{\text{Temporal prior} \\ \text{(Dirichlet-Hawkes)} \\ \downarrow}}$$

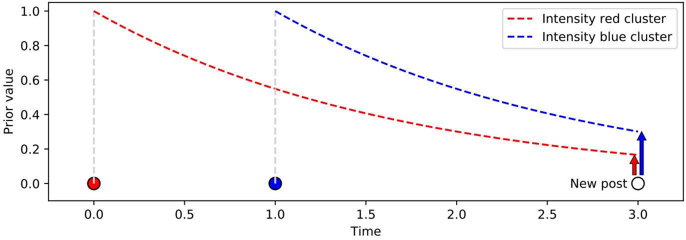


Dirichlet-Hawkes process – Explicit

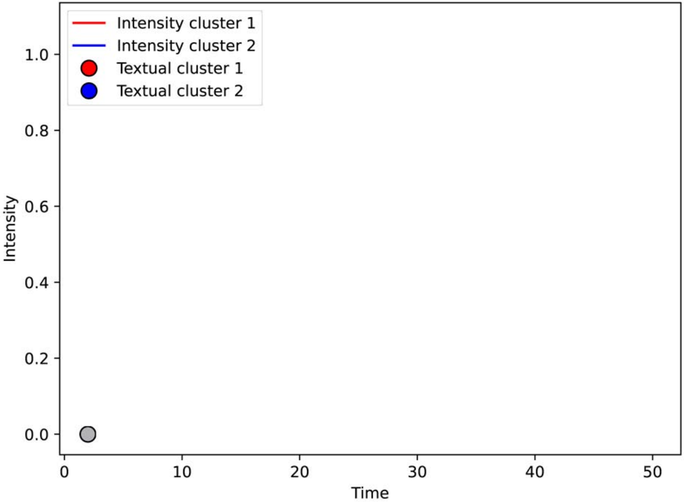
- $P(c|t,\mathcal{H})$: prior probability of cluster c at time t given history $\mathcal{H}$
- $\lambda_c(t)$: intensity of cluster c at time t
- Dirichlet process with counts N_c replaced by $\lambda_c(t)$

$$\underbrace{P(c|t,\mathcal{H})}_{\substack{\text{Temporal prior}\\ \text{(Dirichlet-Hawkes)}}} = \begin{cases} \frac{\lambda_c(t)}{\alpha_0+\sum_k \lambda_k(t)} & \text{if c = 1, ..., K} \\ \frac{\alpha_0}{\alpha_0+\sum_k \lambda_k(t)} & \text{if c = K+1} \end{cases}$$

$\downarrow$



Inference (1 particle)



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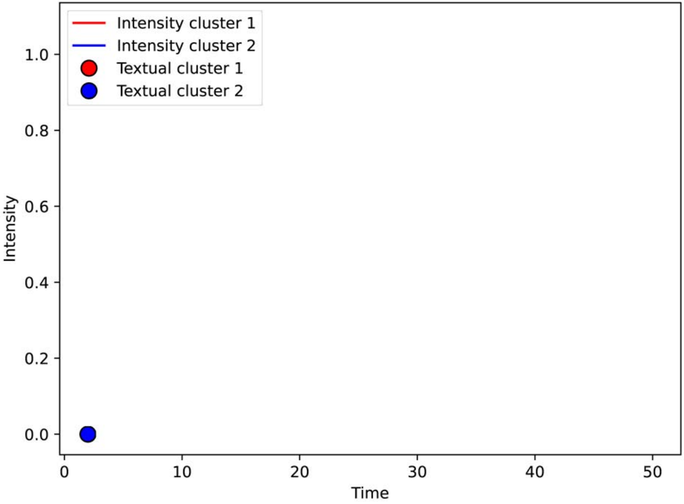

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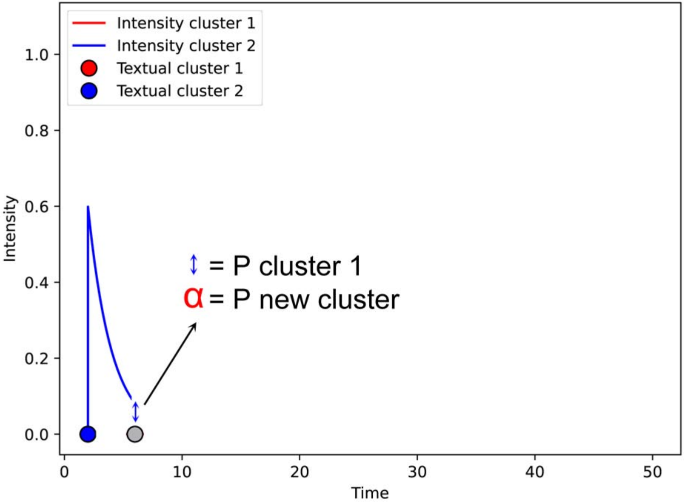

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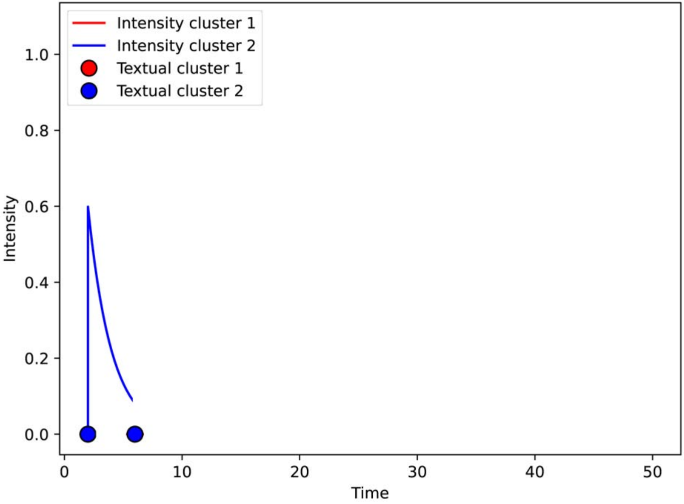

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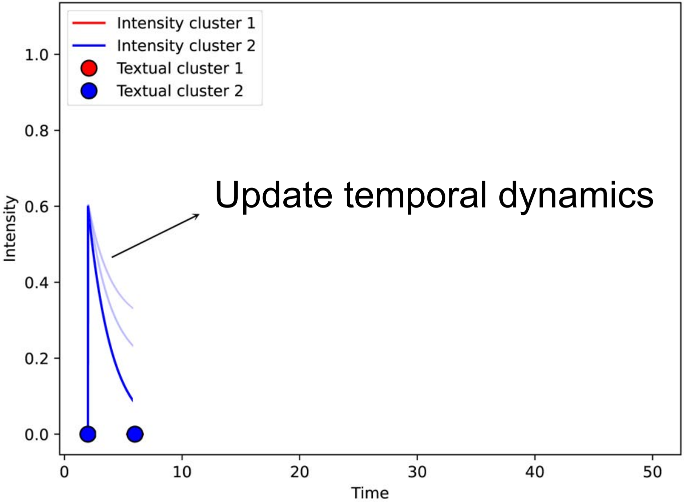

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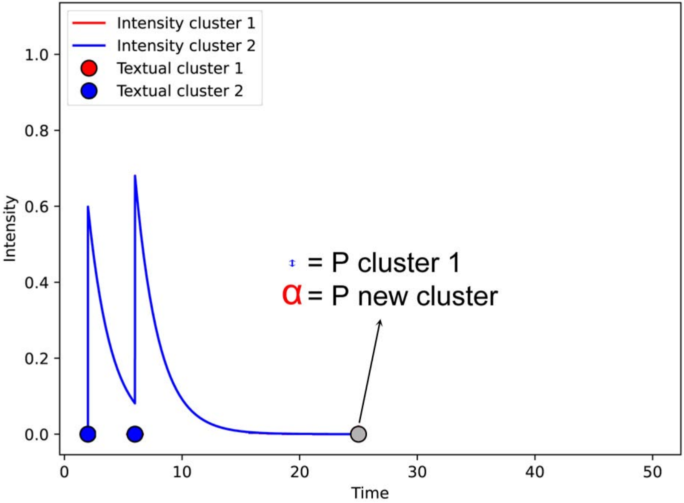

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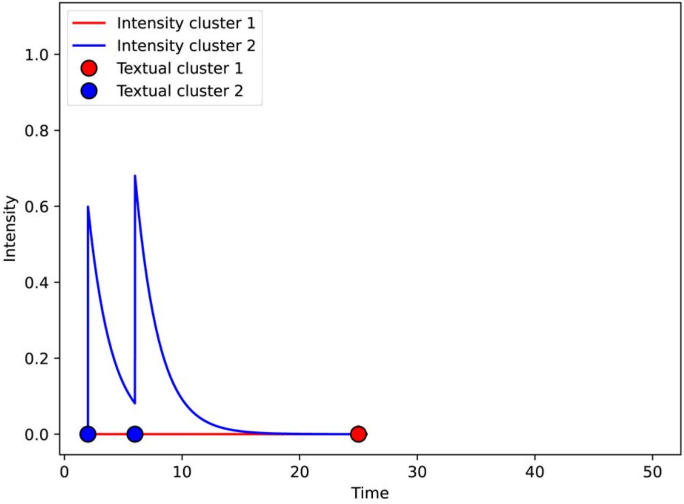

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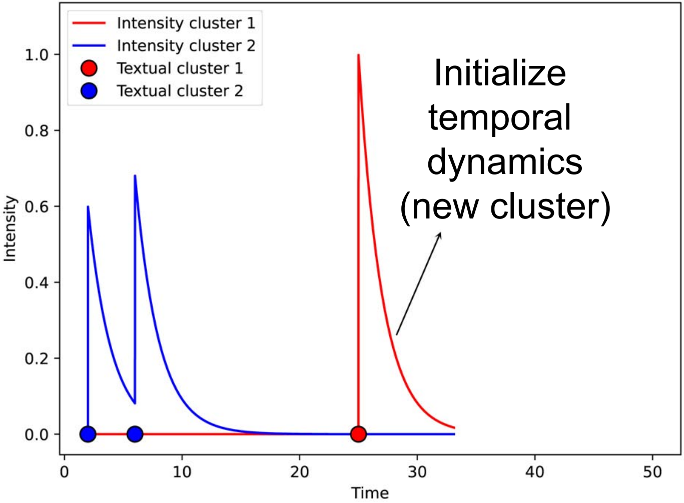

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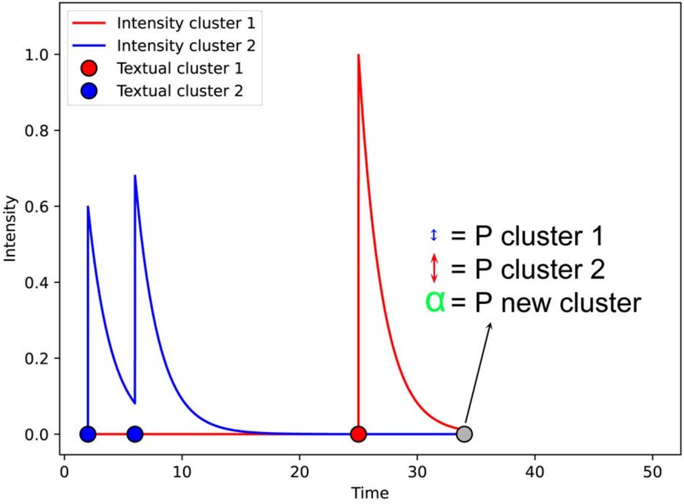

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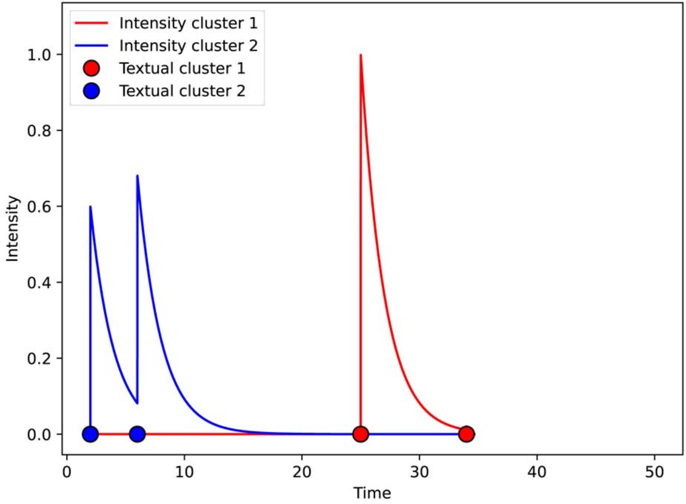

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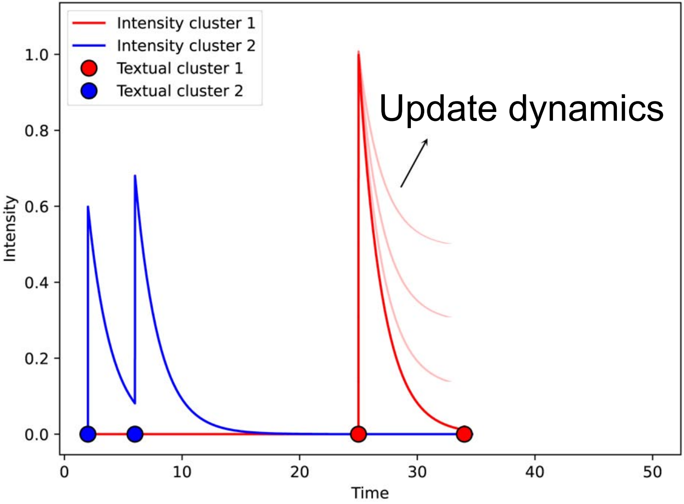
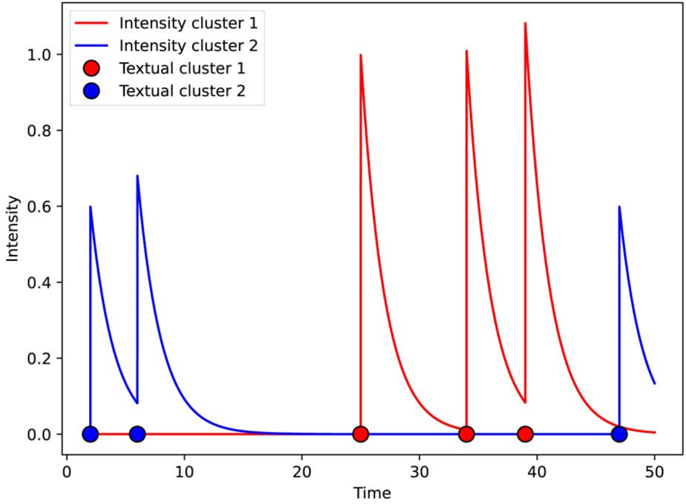
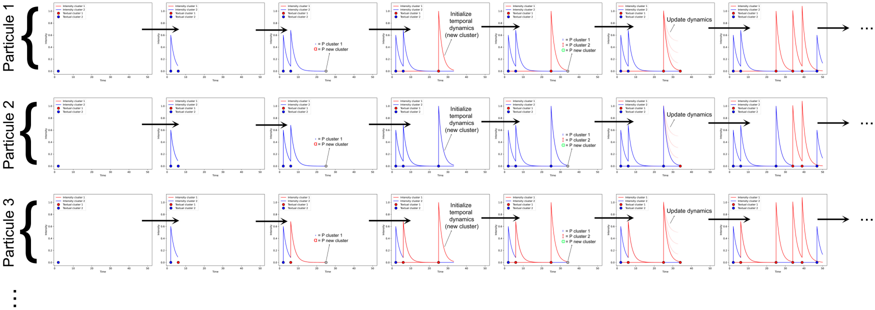


Inference (1 particle)



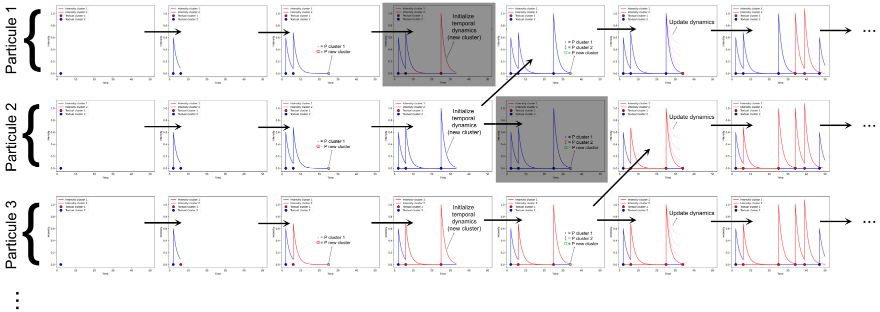
Inference (all particles)

- Run simultaneously on several *particles*

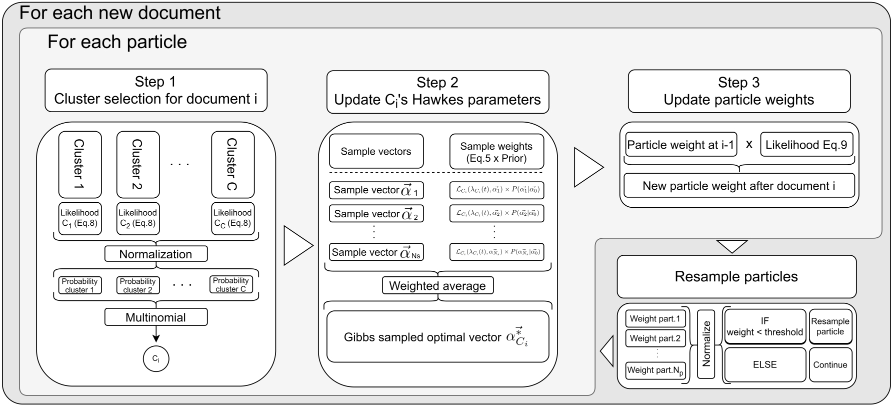


Inference (all particles)

- Discard unlikely particles and replace them by more likely ones



Inference (summarized)



Performances (well-separated)

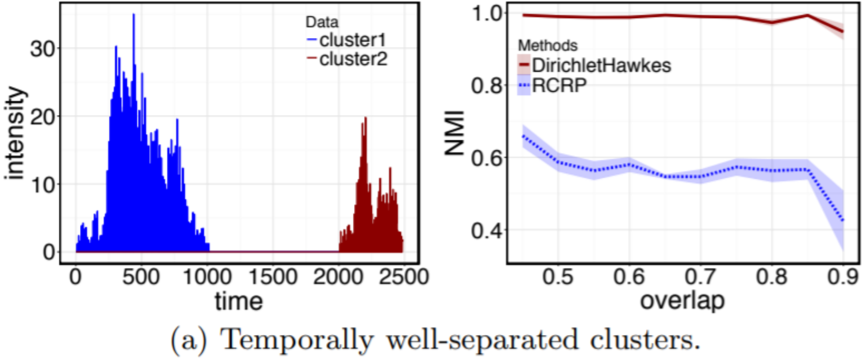


Figure 10: [Du et al., 2015]

Performances ("not" well-separated)

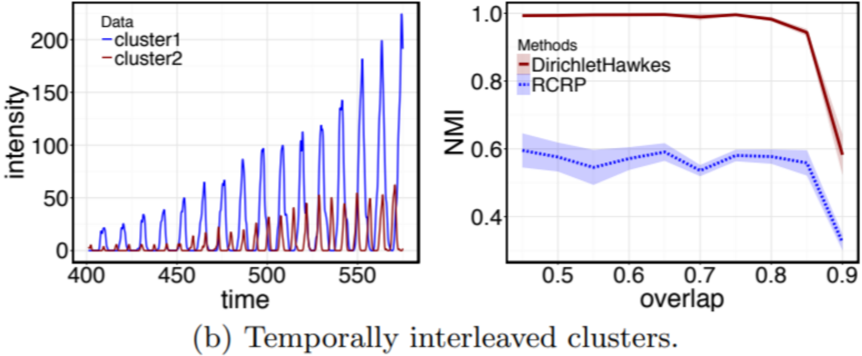


Figure 11: [Du et al., 2015]

Output

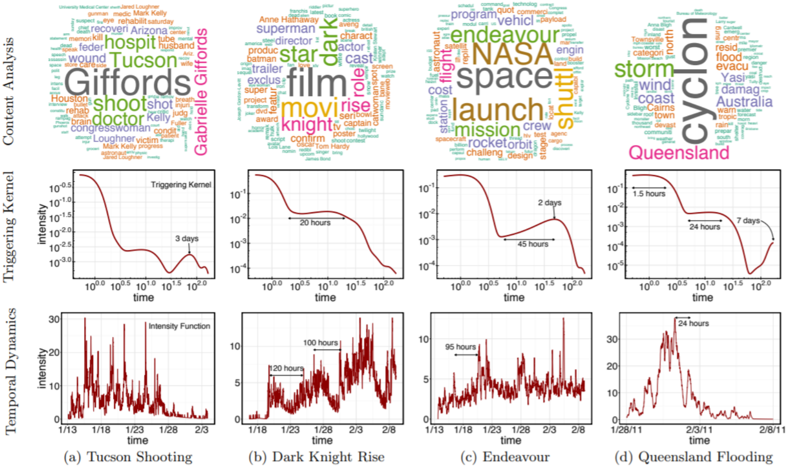


Figure 12: [Du et al., 2015]

Variants

- Numerous variants based on Dirichlet-Hawkes process
  - ◇ Hierarchical (CRF) and Nested (nCRP) extensions of DHP
  - ◇ Multivariate DHP [Zheng et al., 2021]
  - ◇ Not-vanishing DHP prior [Kapoor et al., 2018]

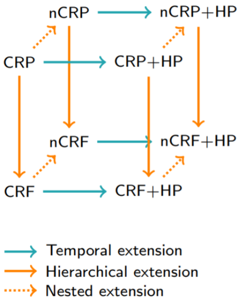


Figure 13: [Kapoor et al., 2018]

BUT!

Dirichlet prior is a choice

- Dirichlet-based priors are an arbitrary choice
  - Other priors are as fit [Welling, 2006]
  - The choice of the prior matters [Wallach et al., 2009]
  - Few variations proposed [Wallach et al., 2010, Pitman and Yor, 1997]
- DP exhibits "rich-get-richer" property
  - Why linear dependence?
  - Why this assumption at all? [Wallach et al., 2010]

Powered Dirichlet process

- Powered Chinese Restaurant Process:

$$PCRP(C_i = c | C_1, ..., C_{i-1}, \alpha, r) = \begin{cases} \frac{N_c^r}{\alpha + \sum_k N_k^r} & \text{if c = 1, ..., K} \\ \frac{\alpha}{\alpha + \sum_k N_k^r} & \text{if c = K+1} \end{cases}$$

- ◇ r<0: "rich-get-poorer"
- ◇ r=0: "rich-get-no-richer" (Uniform Process)
- ◇ 0<r<1: "rich-get-less-richer"
- ◇ r=1: "rich-get-richer" (Dirichlet Process)
- ◇ $r = \frac{\log(N_k - \beta)}{\log(N_k)}$: "rich-get-richer" (Pitman-Yor Process)
- ◇ r>1: "rich-get-more-richer"

PDP impact

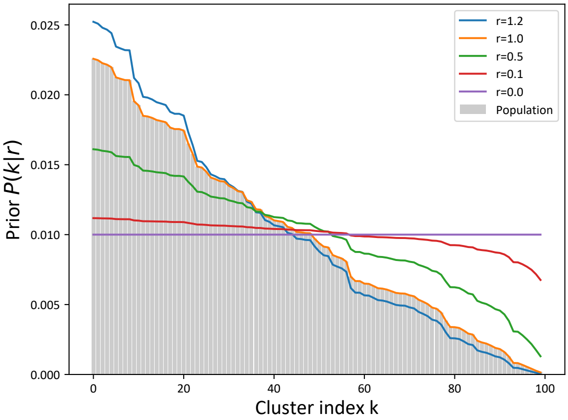
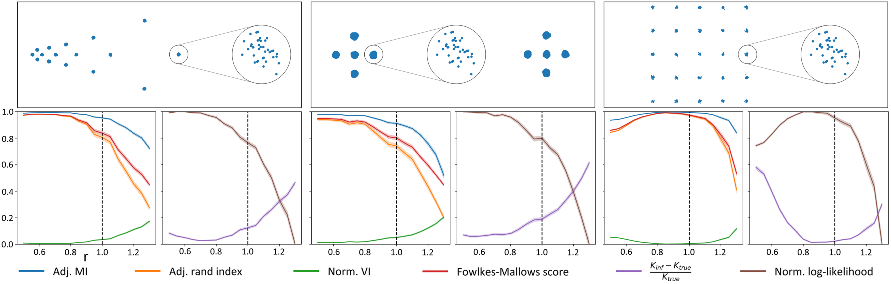


Figure 14: Prior probability for each of 100 clusters whose population is known (grey bars) w.r.t. r

Results

- Use as prior for IGMM
- DP not always the best prior



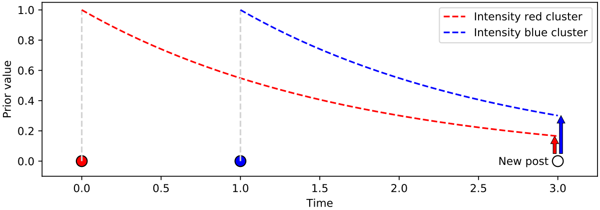
PDP into DHP

- Powered priors: controlling the informativeness of the prior
  - ◇ PDP: strength of the "rich-get-richer" hypothesis
  - ◇ PDHP: strength of the temporal dependence hypothesis
- PDHP [Poux-Médard et al., 2021]:

$$\underbrace{P(c|t,\mathcal{H},r)}_{\text{PDHP prior}} = \begin{cases} \frac{\lambda_c(t)^r}{\alpha_0+\sum_k \lambda_k(t)^r} & \text{if c = 1, ..., K} \\ \frac{\alpha_0}{\alpha_0+\sum_k \lambda_k(t)^r} & \text{if c = K+1} \end{cases}$$

- Generalization:
  - ◇ Uniform process: $r = 0$ (only textual information)
  - ◇ Dirichlet-Hawkes process: $r = 1$ (temporal and textual information)
  - ◇ Deterministic Hawkes process: $r \to \infty$ (only temporal information)

Effect of r



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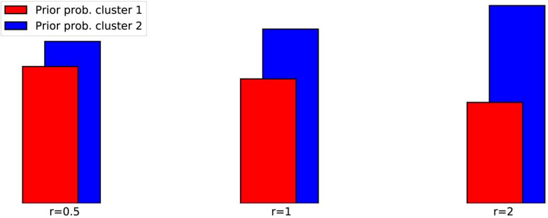


Changes induced by PDHP

$$P(\text{cluster}|\text{text, time}) \propto \underbrace{P(\text{text}|\text{cluster})}_{\text{Textual likelihood}} \times \underbrace{P(\text{cluster}|\text{time, r, history})}_{\text{PDHP temporal prior}}$$

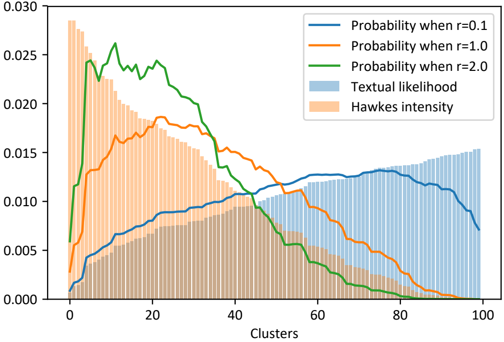
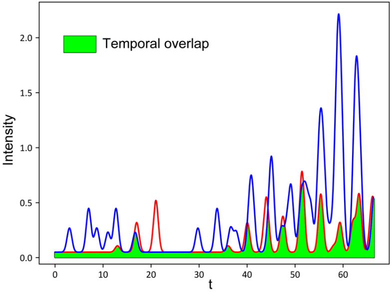


Figure 15: [Poux-Médard et al., 2021]

Why is it relevant - Overlaps

- Often, a piece of information is more informative than the other:
  - ◇ Twitter: short texts (few textual information) but informative cascade dynamics (helpful temporal information)
- Happens often because of overlaps:



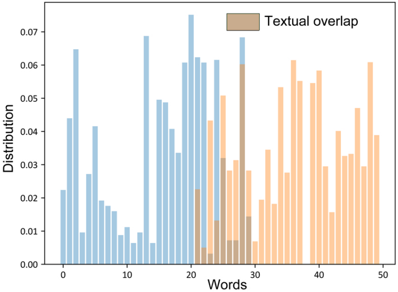


Figure 16: [Poux-Médard et al., 2021]

Results for various overlaps

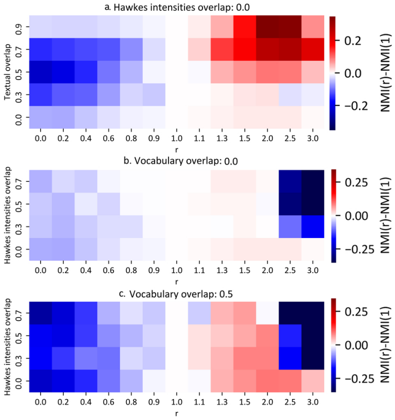
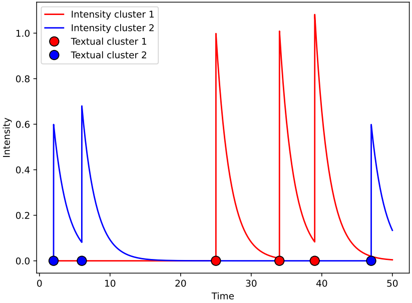


Figure 17: [Poux-Médard et al., 2021]

- PDHP adapts to various situations better than DHP:
  - Large textual overlap
  - Large temporal overlap
  - No overlap
- Up to +0.3 NMI in our case

Why is it relevant - Decorrelations

- Decorrelations:
  - Ex: influent journal publishing on a topic does not have same dynamics as less influent one on the same topic



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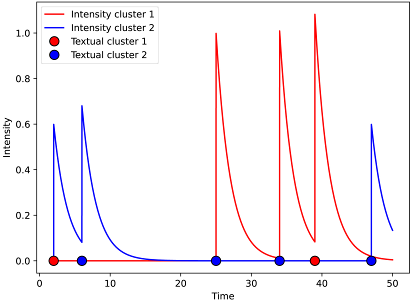


Results for various decorrelations

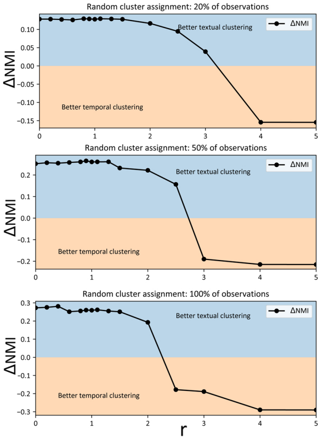


Figure 18: [Poux-Médard et al., 2021]

- PDHP retrieves either temporal or textual clusters
  - Small r: good textual clusters
  - Large r: good temporal clusters

Reddit r/news - Typical output

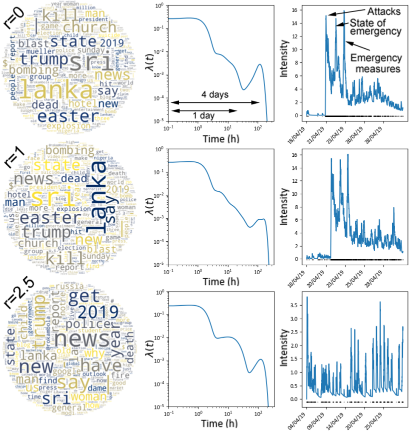


Figure 19: [Poux-Médard et al., 2021]

- Real world data: r/news
- Different clusters and dynamics for different r
  - Small r: similar vocabulary
  - Large r: specific dynamics

Reddit r/news, r/TodayILearned, r/AskScience - Some metrics

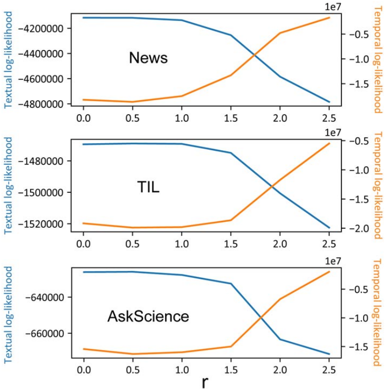


Figure 20: Textual and temporal likelihood vs r [Poux-Médard et al., 2021]

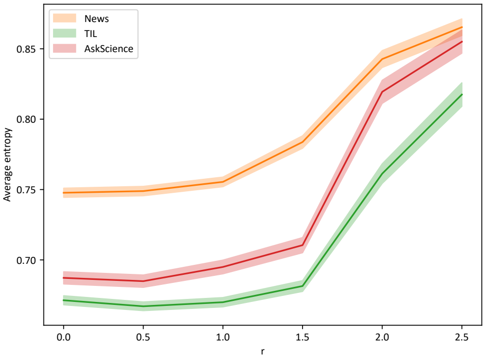


Figure 21: Entropy of textual clusters: sharper textual clusters for low r [Poux-Médard et al., 2021]

Structure matters!

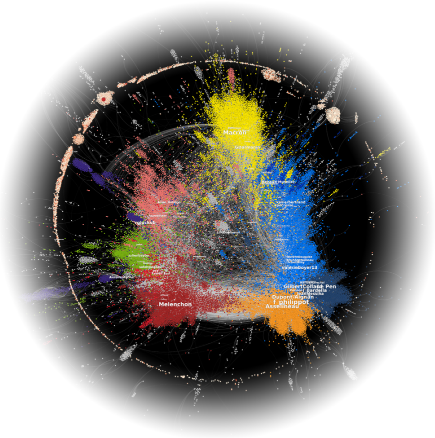


Figure 22: A sample from the Twitter structure (Politoscope [Gaumont et al., 2018])

Why (P)DHP is incomplete

- DHP prior accounts for time but not structure
  - ◇ Infers aggregated dynamics
  - ◇ Misses the structural aspect: discussions are not the same among different groups

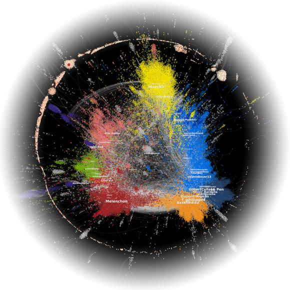
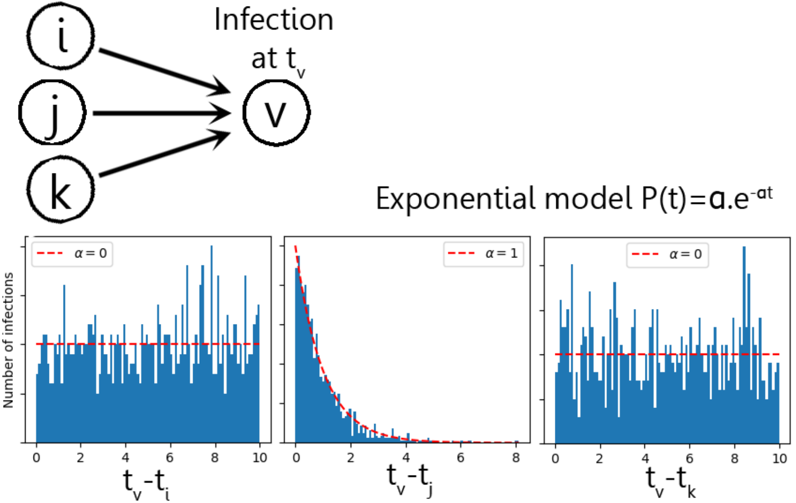


Figure 23: A sample from the Twitter structure (Politoscope [Gaumont et al., 2018])

Network inference



Network inference – Literature

- Several works on network inference using survival analysis:
  - ◇ NetRate [Gomez-Rodriguez et al., 2011]
  - ◇ InfoPath [Gomez-Rodriguez et al., 2013a]
  - ◇ KernelCascade [Du et al., 2012]
  - ◇ MoNet [Wang et al., 2012]
  - ◇ TopicCascade [Du et al., 2013]
- They are all special cases of [Gomez-Rodriguez et al., 2013b]
  - ◇ Bridges the gap between survival analysis and point processes
  - ◇ Formulates each of previous models as a counting point process

Point process

- Network inference naturally embeds into point processes literature
  → We can derive a temporal *and* structural Bayesian prior

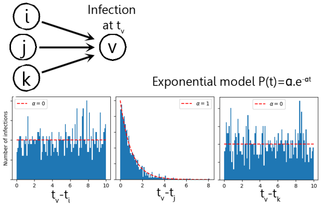


Figure 24: Survival process

Both are

point

processes

$<\approx>$

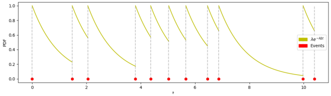


Figure 25: Hawkes process

Temporal and structural prior

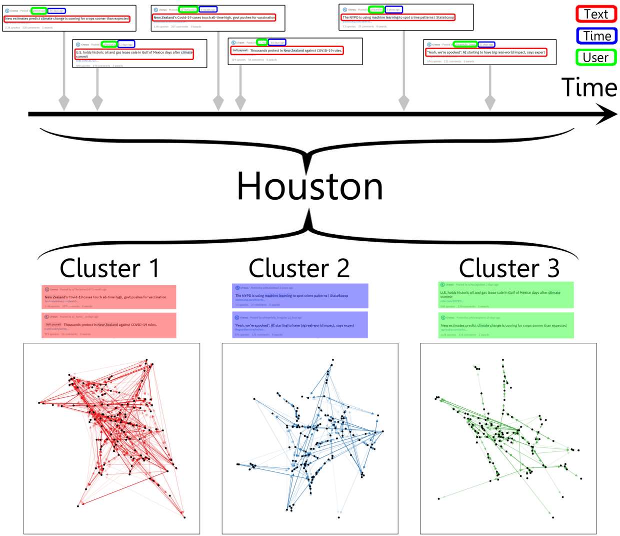
- Houston: **H**eterogeneous **O**nline **U**ser-**T**opic **N**etwork inference
- Prior on cluster membership C_i of observation i observed on node u at time t given history $\mathcal{H}$ and cluster-dependent networks A:

$$P(C_i = k|u, t, \mathcal{H}, A)$$

$$= \begin{cases} \dfrac{\lambda_0^{(k)} + \sum_{\mathcal{H}_{i,c}^{(k)}} H(t_i^c | t_j^c, \alpha_{u_j^c, u_i^c}^{(k)})}{\lambda_0^{(K+1)} + \sum_k^K \lambda_0^{(k)} + \sum_{\mathcal{H}_{i,c}^{(k)}} H(t_i^c | t_j^c, \alpha_{u_j^c, u_i^c}^{(k)})} & \text{if k = 1, ..., K} \\ \dfrac{\lambda_0^{(K+1)}}{\lambda_0^{(K+1)} + \sum_k^K \lambda_0^{(k)} + \sum_{\mathcal{H}_{i,c}^{(k)}} H(t_i^c | t_j^c, \alpha_{u_j^c, u_i^c}^{(k)})} & \text{if k = K+1} \end{cases}$$

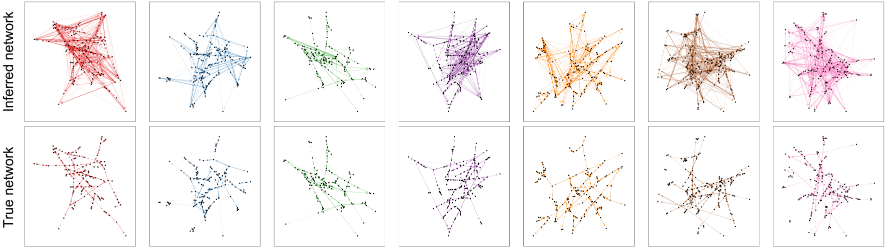
$$= \begin{cases} \dfrac{\text{Strength of incoming edges of cluster/subnetwork k at time t}}{\text{Normalizing term}} & \text{if k = 1, ..., K} \\ \dfrac{\text{Probability of a new cluster/subnetwork k+1 at time t}}{\text{Normalizing term}} & \text{if k = K+1} \end{cases}$$

Task



Results – Synthetic

- We simulate the spread of documents drawn from 5 topics, each with its own vocabulary and subnetwork

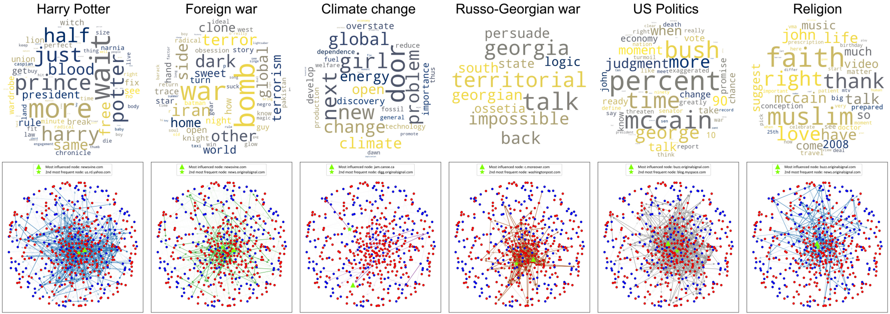


Numerical results

		Houston	TC	DHP	NetRate
PL	NMI	**0.809**	0.669	0.449	-
	ARI	**0.688**	0.330	0.063	-
	AUC	**0.807**	0.719	-	0.731
	MAE	**0.267**	0.338	-	0.460
ER	NMI	**0.787**	0.711	0.638	-
	ARI	**0.631**	0.488	0.411	-
	AUC	**0.849**	0.800	-	0.659
	MAE	**0.229**	0.278	-	0.481
Blogs	NMI	**0.750**	0.668	0.372	-
	ARI	**0.609**	0.365	0.023	-
	AUC	0.701	0.613	-	**0.710**
	MAE	**0.374**	0.444	-	0.499

Results – Real world

- Memetracker data (2009)



Conclusion

- Dirichlet and Hawkes process have an old and separate history
  - ◇ Only recently (2015) they have been brought together
  - ◇ Their reunion launched a new branch of inductive machine learning
- The number of extensions based on Dirichlet-Point-Processes might be enormous, because we touched core concepts of machine learning
  - ◇ Dirichlet processes (PDP): could be used to redefine hierarchical DP, nested DP, or any models built on them (LDA, SBMs, among others)
  - ◇ Point processes (Poisson, Hawkes, Survival/Counting, etc.): the new possibility to merge them with DP could lead to a potentially infinite number of different Dirichlet-Point-Process priors.
- We presented 2 of such extensions:
  - ◇ PDP+HP $\rightarrow$ PDHP (flexible temporal prior)
  - ◇ DP+Survival $\rightarrow$ Houston (temporal+structural prior)

Thanks for your attention!

(**DP**, HDP, nHDP, **PDP**, IBP, PIBP, PnHDP, PPY, PnPY, PHPY, ...)

$\times$

(**Hawkes**, **Survival**, Cox, Poisson, Determinantal, Geometric, ...)

$=$

(**DHP**, HDHP, IBHP, **PDHP**, **Houston**, ...?)

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